SEMI-EMPIRICAL MODEL FOR TACHYCARDIA ORIGINATED FROM FITZGHEW-NAGUMO EQUATIONS

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Abstract Spontaneous fluctuations that can be observed in cardiac dynamics, like tachycardia, are very dangerous, and may case cardiovascular collapse, cardiac arrest and death. The problem is extremely complicated, as result of huge number of influencing factors, when their role has not been adequately studied. It this context some researchers are trying to use ideas borrowed from chaos theory for a more in-depth understanding of tachycardia. However, this approach requires a cautious attitude for the following reason. It is known that one of the characteristic features associating with the transition from regular oscillations to chaotic ones is the doubling of their period. It means that in chaotic processes, the frequency of oscillations decreases with time. So, for the equal time periods there are less oscillations than during the regular regime. But the main fact of tachycardia is, as we know, that the number of its oscillations is sharply increasing. Such heart rhythms, especially those that are long enough to affect the heart's function, can be deadly.The goal of present work is to provide an appropriate mathematical description for the tachycardia phenomenon without using ideas from the chaos theory. It should take into account the main property of tachycardia: an increasing in oscillations number per same time period in comparison with the case of a healthy heart. For obtaining the goal, we use the semi-empirical model originated from the Fitzhugh-Nagumo system of two coupled ordinary differential equations.

Index Terms activator, Fitzghew_Nagumo , inhibitor ,heart-beats, mathematical description of tachycardia, numerical model, oscillations.

1 Introduction

Heart arrhythmias remain the main scourge of mankind . Typical examples of abnormal heart rhythms are tachycardia, bradycardia and chaotic rhythm, called atrial or ventricular fibrillation. Arrhythmias produce a board range of symptoms from barely noticeable to cardiovascular collapse, cardiac arrest and death [1].

There is suggested in these models that the excitation of cardiac cells is initiated by a sudden change in electrical potential across the cell membrane due to the transmembrane flux of charged ions. The first model of the excitation by controlled opening and closing channels was proposed by Alan Lloyd Hodgkin and Andrew Huxley [2]. The Hodgkin Huxley (HH) model was simplified to obtain Fitzhugh-Hagumo (FHN) model [3]. It allowed full qualitative analysis of underlying action potentials by mathematical tools. The model was formulated in terms of fast action potential and slow recovery variable..

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2 Further developments in the Mathematics and Physics of the Heart oscillations

The numerical methods for solving the corresponding mathematical problems were improved. In particular, explicit and implicit finite-difference schemes, the finite elements and Runge-Kutta methods within MATLAB computational environment were successfully used.

The physical principles underlying background HH and FHN models were clarified in a number of works, which deal with ,for examples travelling excitation waves, spiral wave breaking phenomena[4], Eikonal equations [5] and some ideas originated in the chaos theory [6] with regards to the periods doubling effect.

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In addition, there is an interest in the use of ancient Chinese medicine data in the science of the heart.
An example is given in the [7] where problem of the blood flow in the elastic vessels is analyzed with taking into consideration the mapping of the points responsible for humans' organs on the arms surfaces as depicted below in the Fig.1.

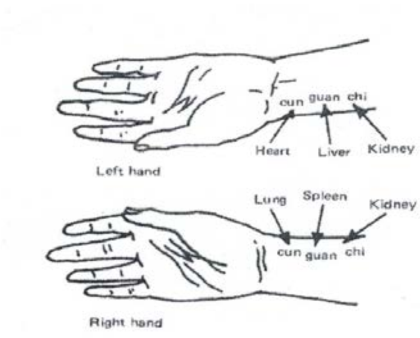

Fig. 1. A distribution of the points responsible on the human's organs on surfaces of the arms [7].

3 Period doubling leading to chaos.

Consider an example from the chaos theory applied to the population dynamics. Below there is a model with two prey species named as R_1, R_2 which are eaten by the predators belonging to the single specie N [6].
Here $a_1, a_2, a_{12}, a_{21}, ca_1, ca_2$ are the set of the parameters characterizing the model. Equations were solved numerically and results are depicted in the Fig.2

$$
(1)\quad \begin{aligned}
R_t' &= R_1(1 - R_1 - a_{12}R_2) - a_1 R_1 N \\
R_t' &= R_2(1 - R_2 - a_{21}R_1) - a_2 R_2 N \\
N_t' &= N(ca_1 R_1 + ca_2 R_2 - 1)
\end{aligned}
$$

In the Fig.2. there are graphs presenting sequential solutions for populations R_1, R_2 and N vs time t from periods 0-100,0-200,0-500 units. The doubling cascade arrives .It means that the later time oscillations are needing more time to be performed with comparison to the early time oscillations. It seems that systems similar to that described in the Fig.2 indeed could be used with relation to modeling bradycardia. But it isn't possible describe in such a way phenomenon of tachycardia when durations of the oscillations are diminishing with regards to the normal-state heart beats. Thus transfer ideas from the chaos to heart problems needs to be made with caution.

4.Governing Equations

In this work we use the Fitzhugh-Nagumo (FHN) system consisting of two ordinary differential equations describing the coupled oscillations of the activator and inhibitor variables in the presence of four empirical coefficients. FHN system is a simplified version of the more complicated HH (Hodgkin-Huxley) model consisting of four ordinary differential equations and a large number of empirical constants [4]. The model of tachycardia presented in this work

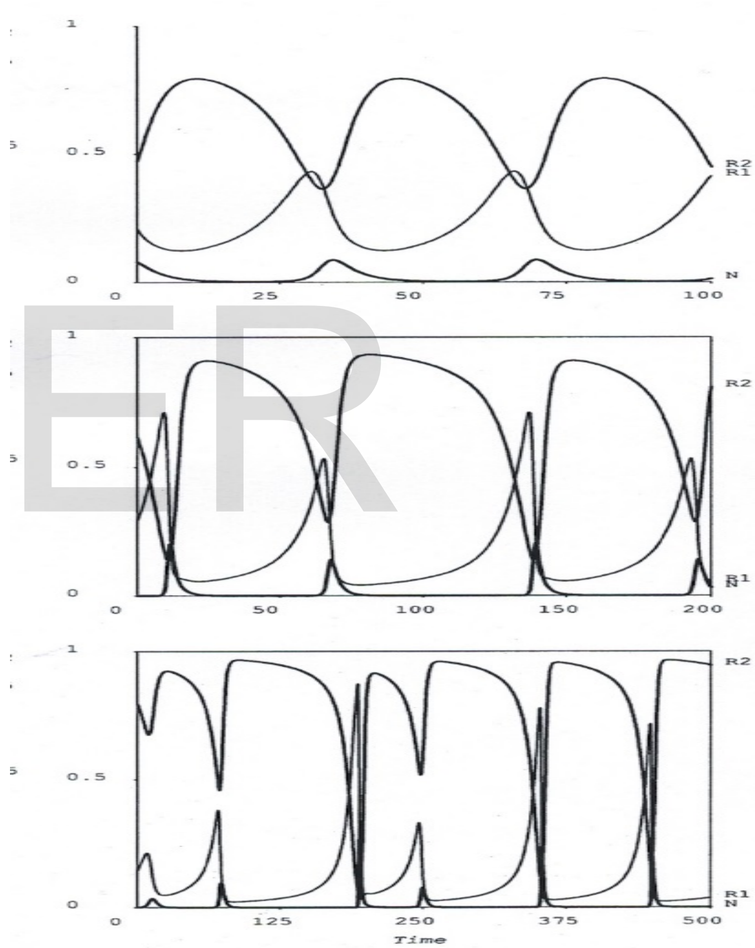

Fig.2 Solution of "Two preys and single predator species vs time problem".

operates with the parameters from the Fitzhugh-Nagumo model. The governing equations are written as follows:

$$
(2)\quad x' = c(y + x - x^3/3 + f)
$$

$$y' = -(x - a + by)/c$$

Here x, y are respectively fast and slow variables named as Activator and Inhibitor for the sake of convenience; a, b, c, f are parameters. In this notation f is characteristic of the stimulus intensity, while a, b, c usually are given without mentioning on their physical meanings. The goal is to integrate and obtain an insight of the effect of the above parameters on oscillations with regards to fibrillation. As an integration tool was chosen Matlab-2013 package and its ode 45 solver to integrate ordinary differential equations by means of the classical Runge-Kutta finite-difference method.

5 Results

Below are described three hypothetical scenarios with normal, rapid and ultra-rapid heart-beats. Normal heart-beats hypothetically characterize the healthy heart and are governed by the following FHN parameters : a =0.1,b=0.5, c=10 and f=1.

5.1. Scenario 1:normal heart-beats

The normal heart-beats are obtained by means of the application of the Runge-Kutta method for the integration of the FHN equations using above mentioned parameters.Fig.1 shows the combined graphs for the dimensionless activator u_1 and the inhibitor v_1 as a functions of the dimensionless time t. It follows from the Fig.3 that for 100 units of time, for the given parameters a, b, c, f four oscillations were produced for the normal rhythm of the heart-beats.

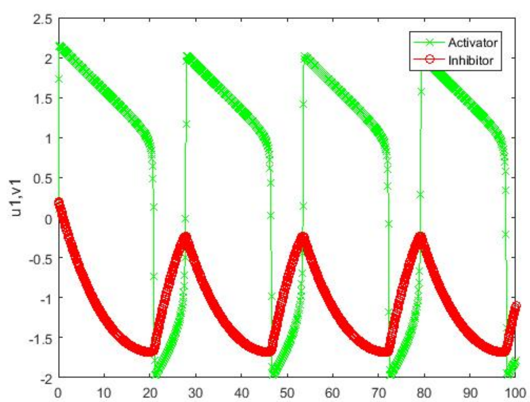

Fig.3. Solution for the the activator $u1$ and inhibitor $v1$ for normal heart-beats governed by the undisturbed FHN equations.Here $u1, v1$ are the activator and inhibitor for $a = 0.1, b = 0.5, c = 10, f = 1$.

5.2. Scenario 2:rapid heart-beats

In the 2-nd Scenario consider a case with disturbed parameters defined as follows:
$a = 0.1, b = 0.03, c = 10, f = 1$. It means that a, c, f are taken as in the Scenario 1 and parameter b is essentially diminished. Results of the FHN equations solution are depicted in the Fig.4 and Fig.5.

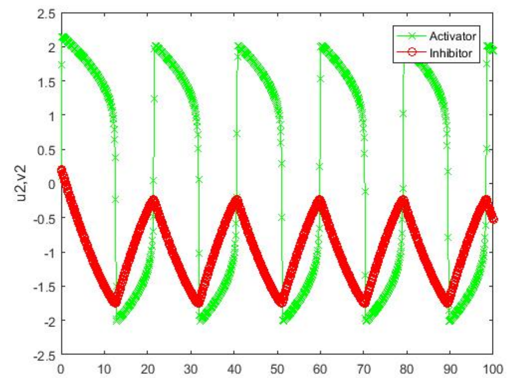

Fig.4. Solution for the the activator $u2$ and inhibitor $v2$ for the rapid heart-beats governed by the disturbed FHN equations . Here $a = 0.1, b = 0.03, c = 10, f = 1$.

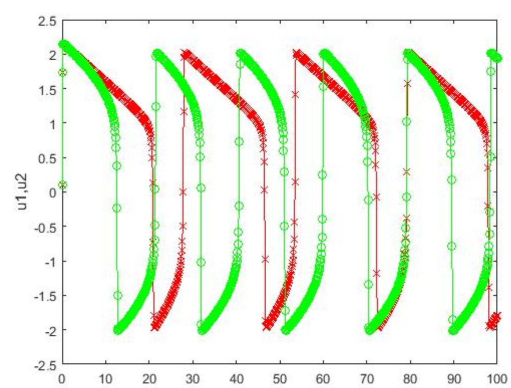

Fig.5. A comparison between the activators u1 and u2 vs time for the normal and rapid heart-beats governed by the undisturbed and disturbed FHN equations with parameters $a = 0.1, b = 0.5, c = 10, f = 1$. and $a = 0.1, b = 0.03, c = 10, f = 1$.

It may be concluded from the Fig.4 and Fig.5 that for the same time there are four heart- beats at the undisturbed conditions and almost six for disturbed conditions. These rapid beats are the main attribute of the tachycardia in the adopted model.

5.3. Scenario 3: ultra-rapid beats.

The governing parameters for the FHN equations solution were taken as $a = 0.1, b = 0.03, c = 2, f = 1.$ It means that parameter c also is changed with comparison to its value from the Scenario 2. Results of the numerical integration are depicted in the Fig.6 and Fig.7.

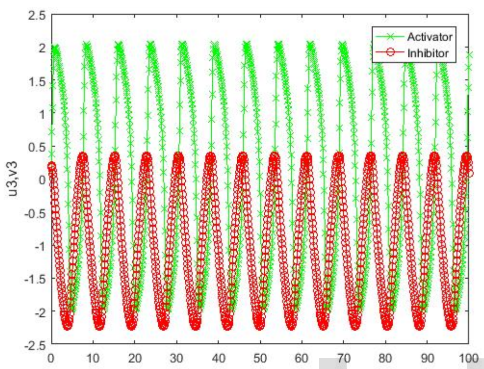

Fig.6. Solution for the activator u3 and inhibitor v3 vs time for ultra-rapid heart-beats governed by the disturbed FHN equations with $a = 0.1, b = 0.03, c = 10, f = 1.$

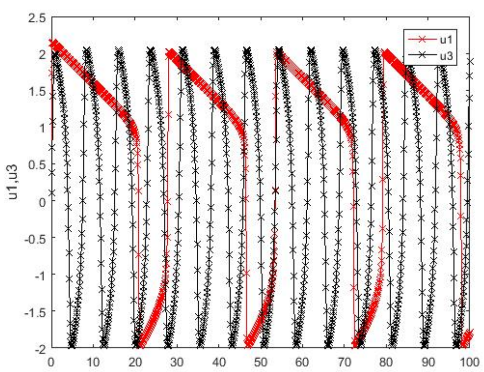

Fig.7. Solutions for the activator u1 and activator u3 vs time for the normal and ultra-rapid heart-beats for undisturbed and disturbed FHN equations with a=0.1,b=5,c=10,f=1 and a=0.1,b=0.03,c=2,f=1 .

It follows from these Figures that for 100 units of time 13 oscillations arrive, which corresponds to the ultra-rapid rhythm of the heart-beats. Thus in the ultra-rapid case the number of oscillations is three times greater than normal, and this is already incompatible with human life within the framework of the adopted model.

6. **Conclusions**

From the present work, the following conclusions can be drawn.

1. Chaotic dynamics can be useful in analyzing processes such as bradycardia because the cascade of period doubling is physically manife-sted in the slowing down of heart rhythms.

2.Correspondingly, with tachycardia which is characterized by increasing heart rhythms the manifestations of a cascade of period doubling can lead to arrival of misunderstandings that do not correspond to the physical meaning of the process being studied.

3.The possibility of reproducing the phenomenon of tachycardia in the conditions of a computer experiment can be realized within the frame-work of the presented semi-empirical model originated from the FHN model empirically by variation of the determining constants named a,b,c,f. Three hypothetical Scenarios leading to the normal,rapid and ultra-rapid beats are considered.A conclusion was that ultra-rapid beats are incompatible with life.

4.Also there is of interest to perform numerical simulation of the hearts rhytms taking into consideration ancient chinese mapping on the humans arms the points responsible for humans body organs ,e.g. the heart.

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